

## 2. Dimensional Analysis and Hydraulics - Similitude

(1x3) (i)

\* Dimensional homogeneity :- Dimensional homogeneity is said to be

An equation is called dimensionally homogeneous if the fundamental dimensions have identical powers of  $M, L, T$  on both sides.

An equation is said to be dimensionally homogeneous if the dimensions of the terms on its left hand side are same as the terms on its right hand side.

- Let us consider the eqn:  $V = \sqrt{2gH}$ .

$$LT^{-1} = \sqrt{1 \times LT^{-2} \times L}$$

$$\boxed{LT^{-1} = LT^{-1}}$$

- Dimensions of LHS =  $(V) = LT^{-1}$

- Dimensions of RHS =  $\sqrt{2gH} = \sqrt{1 \times LT^{-2} \times L}$

$$= LT^{-1}$$

$\therefore$  Dimensions of LHS = Dimensions of RHS.

- Equation  $V = \sqrt{2gH}$  is dimensionally homogeneous.  
So it can be used for any system of units.

Another example :-

$$Q = AV$$

$$Q = L^3 T^{-1}$$

$$A = L^2, V = L T^{-1}$$

$$Q = A \times V$$

$$L^3 T^{-1} = L^2 \times L T^{-1}$$

$$L^3 T^{-1} = L^3 T^{-1}$$

$\therefore$  It is homogeneous. UNIT-3, Pg-1 / 29

$$\text{eg: } V = c\sqrt{mi}$$

$$V = (1) (L \times 1)^{\frac{1}{2}}$$

$$V = LT^{-1}, \sqrt{mi} = L^{\frac{1}{2}}$$

$$LT^{-1} \neq L^{\frac{1}{2}}$$

$\therefore$  It is non-Homogeneous

### \* Method of Dimensional Analysis:-

If the no. of variables involved in a physical phenomenon are known, then the relation among the variables can be determined by the following 2 methods.

(i) Rayleigh's method

(ii) Buckingham's pi method.

(i) Rayleigh's Method! -

This method is used for determining the expression for a variable which depends upon max 3 or 4 variables only.

$\rightarrow$  If the no. of independent variables becomes more than 4, then it is difficult to find the expression of Dependent Variable.

$\rightarrow$  Let 'x' is variable, which depends on  $x_1, x_2, x_3$  variables.

Then According to Rayleigh's method 'x' is function of  $x_1, x_2, x_3$  and Mathematically it is written as

$$x = f(x_1, x_2, x_3)$$

This can also be written as  $x = k x_1^a x_2^b x_3^c$ .

where

k is constant, and

a, b, c are arbitrary powers,

- The value of a, b, c are obtained by comparing the powers of fundamental dimension on both sides.
- Thus the expression is framed for dependent variable.

The Rayleigh's method is based on three steps:-

1. First of all, write the functional relationship with the given data,
2. Now write the egn in terms of a constant with the exponents  $a, b, c$ .
3. With the help of the principles of dimensional homogeneity, find out the values of  $a, b, c$  by obtaining simultaneous egn.
4. Now substitute the values of these exponents in the main egn and simplify it.

Problem:-

- 1) The time period ( $T$ ) of a pendulum depends upon the length ( $L$ ) of the pendulum and acceleration due to gravity ( $g$ ) derive the expression for the time period.

A) Given data,

- 1) time period ( $T$ ) is depends on length ( $L$ ) & Acceleration due to gravity ( $g$ ).

$$T = f(L, g)$$

$$T = k L^a g^b T^c = k L^a (LT^{-2})^b$$

Substitute the dimensions on the L.H.S.

$$T = k L^a (LT^{-2})^b$$

Equating the powers of M, L, T on both sides

$$\text{power of } T \Rightarrow -1 = -2b$$

$$b = -\frac{1}{2}$$

$$\text{Power of } L \Rightarrow 0, a+b = 0$$

$$\text{sub } b \text{ in eqn } ②. \therefore a = \frac{1}{2}$$

$$\text{from eqn } ②. \therefore a = -b \text{ in terms of } a \text{ & } b$$

$$a = -(-\frac{1}{2}) \therefore a = \frac{1}{2}$$

$$a = \frac{1}{2}$$

Sub a, b values in eqn ①

$$\text{New dimensionless } t = k L^{\frac{9}{2}} g^{\frac{b}{2}}$$

$$t = k L^{\frac{9}{2}} g^{-\frac{1}{2}}$$

From experiment  $k = 2\pi$

$$t = 2\pi \sqrt{\frac{L}{g}}$$

\* Buckingham's +  $\pi$  ( $\Pi$ ) - Theorem :-  
The rayleigh method of dimensional analysis becomes more difficult if the variables are more than the no. of fundamental dimensional (m, L, T). These difficult is overcomes by using Buckingham's  $\Pi$ -theorem.

Statement :-

If there are 'n' variables in a physical phenomenon and if these variable contain 'm' fundamental dimensions. Then the variables are arranged into  $(n-m)$  dimensionless term. Each term is called  $\Pi$ -term.

- Let  $x_1, x_2, x_3, \dots, x_n$  are the variables involved in a physical problem. Let ' $x_1'$  be the independent variables and  $x_2, x_3, x_4, \dots, x_n$  are the independent variable on which  $x_1$  depends then  $x_1$  is a function of  $x_2, x_3, \dots, x_n$  and mathematically it expressed as

$$x_1 = f(x_2, x_3, x_4, \dots, x_n) \rightarrow ①$$

It can also be written as  $f(x_1, x_2, x_3, \dots, x_n) = 0 \rightarrow ②$ .

→ Egn ② is a dimensionally homogeneous egn if contains 'n' variables. If there are 'm' fundamental dimension then, according the buckingham's  $\Pi$ -theorem egn(2) becomes can be written in terms of no. of dimension less groups (or)  $\Pi$ -terms in which no. of  $\Pi$ -terms =  $(n-m)$  Hence egn ② becomes as

$$f(\Pi_1, \Pi_2, \dots, \Pi_{n-m}) = 0 \rightarrow ③$$

2) Each of term is dimensionless and is independent of the system. Each  $\pi$  term contains  $(m+1)$  variable where 'm' is the no. of fundamental dimensions and it is also called repeating variables let, in the above case  $x_2, x_3$  &  $x_4$  are repeating variable, if the fundamental dimensions, m (M, L, T) is 3 then, each  $\pi$ -term is written as.

$$\pi_1 = (x_2^a, x_3^b, x_4^c, x_1) \rightarrow ④$$

$$\pi_2 = (x_2^{a_1}, x_3^{b_1}, x_4^{c_1}, x_5)$$

$$\pi_{n-m} = (x_2^{a_{n-m}}, x_3^{b_{n-m}}, x_4^{c_{n-m}}, x_n)$$

Each eqn is solved by the principle of dimensional homogeneity & value of a, b, c etc are obtained.

These values are substituted in eqn ④ and values of  $\pi_1, \pi_2, \dots, \pi_{n-m}$  are obtained.

- These values of  $\pi$  are substituted in eqn ③.

The final eqn for the phenomenon is obtained by expressing any one of the ' $\pi$ ' terms as a function of others.

$$\pi_1 \neq 0 (\pi_2, \pi_3, \dots, \pi_{n-m})$$

$$\pi_2 \neq 0 (\pi_1, \pi_3, \dots, \pi_{n-m})$$

## I. Method of selecting repeating variables:-

1. As far as possible, the dependent variables should not be selected as repeated variables.
2. The repeating Variable should be chosen in such away that one Variable contains geometric property Other Variable contains flow property of 3rd variable Contains fluid property.

- a) Variable with geometric properties are length ( $l$ )  
width ( $w$ ) height ( $h$ ) etc, ...
  - b) Variables with flow property are Velocity ( $v$ ),  
acceleration ( $a$ ), angular velocity ( $\omega$ ) etc ...
  - c) Variables with fluid property are dynamic  
viscosity ( $\mu$ ), density ( $\rho$ ).
- 3) The repeating variables selected should not form a dimensionless group.
- 4) The repeating variable together must have the same no. of fundamental dimensions.
- 5) No two repeating variables should have the same dimensions.

In the most of the fluid mechanics problems the choice of repeating variable may be

- 1)  $d, v, p$
- 2)  $l, v, \mu$
- 3)  $d, v, \mu$
- 4)  $l, v, \rho$ .

\* The resisting force ( $R$ ) of a super sonic plane during flight can be considered as depended upon the length of the air craft ( $l$ ), Velocity ( $v$ ), air viscosity ( $\mu$ ), air density ( $\rho$ ) bulk modulus of air  $k$ . Express the functional relationship b/w these variables & the resisting force.

- Resisting force depends on 1) length ( $l$ ) 2) Velocity
- 3) Viscosity ( $\mu$ ) 4) Density ( $\rho$ ) 5) Bulk modulus ( $k$ ).

Total no. of variables ( $n$ ) = 6.

22)

No. of fundamental dimensions ( $m$ ) = 3

Total no. of Dimensions  $\pi$  terms =  $(n-m)$  terms

$\pi$  terms =  $(n-m)$  terms

$$= 6-3$$

$$= 3 \text{ terms.}$$

$$f(\pi_1, \pi_2, \pi_3) = 0 \quad \text{---} \textcircled{1}$$

Each  $\pi$  term contains  $m+1$  variables,

Repeating variables are  $l, v, f$ .

$$R = f(l, v, u, P, k)$$

$$f(R, l, v, u, f, k).$$

$$\pi_1 = [l^{a_1} v^{b_1} f^{c_1} R]$$

$$\pi_2 = [l^{a_2} v^{b_2} f^{c_2} u]$$

$$\pi_3 = [l^{a_3} v^{b_3} f^{c_3} k]$$

$$1. \pi_1 = [l^{a_1} v^{b_1} f^{c_1} R] - \textcircled{2}$$

Substitute dimension on b.s.

$$M^0 L^0 T^0 = L^{a_1} (LT^{-1})^{b_1} (NL^{-3})^{c_1} (MET^{-2})$$

equating the powers of  $M, L, T$  on b.s.

$$\text{Power of } M \rightarrow 0 = c_1 + 1 \Rightarrow c_1 = -1$$

$$\text{Power of } L \rightarrow 0 = a_1 + b_1 - 3c_1 + 1$$

$$\text{Power of } T \rightarrow 0 = -b_1 - 2$$

$$\boxed{b_1 = 2}$$

$$a_1 = -b_1 + 3c_1 - 1$$

$$(a_1 = 2 - 3 - 1) \quad \boxed{a_1 = -2}$$

$\therefore$  sub  $a_1, b_1, c_1$  values in eqn  $\textcircled{2}$

$$\pi_1 = [l^{a_1} v^{b_1} f^{c_1} R]$$

$$\pi_1 = [l^{-2} v^{-2} f^{-1} R]$$

$$\boxed{\pi_1 = \left( \frac{R}{f l^2 v^2} \right)}$$

$$2) \pi_2 = [l^{a_2} v^{b_2} f^{c_2} u] \rightarrow ③$$

substitute the dimensions on b.s

$$M^0 L^0 T^0 = (L^{a_2}) (LT^{-1})^{b_2} (ML^{-3})^{c_2} (ML^{-1} T^{-1}).$$

equating the powers of M, L, T on b.s

$$\text{Power of } M = 0 \Rightarrow c_2 + 1$$

$$\boxed{c_2 = -1}$$

$$\text{Power of } L = +0 \Rightarrow a_2 + b_2 - 3c_2 - 1$$

$$\text{Power of } T = 0 \Rightarrow b_2 - 1$$

$$\boxed{b_2 = -1}$$

$$\Rightarrow a_2 = -b_2 + 3c_2 + 1$$

$$= (-1) + 3(-1) + 1$$

$$\Rightarrow 1 - 3 + 1 = -1$$

$$\boxed{a_2 = -1}$$

substituting  $a_2, b_2, c_2$  values eqn ⑤

$$\pi_2 = [l^{a_2} v^{b_2} f^{c_2} u]$$

$$\pi_2 = [l^{-1} v^{-1} f^{-1} u]$$

$$\boxed{\pi_2 = \frac{u}{lvf}}$$

$$3) \pi_3 = [l^{a_3} v^{b_3} f^{c_3} k] \rightarrow ④$$

substitute the dimension on b.s

$$M^0 L^0 T^0 = (L^{a_3}) (LT^{-1})^{b_3} (ML^{-3})^{c_3} (ML^{-1} T^{-2})$$

equating the powers of M, L, T on b.s

$$\text{power of } M = 0 \Rightarrow c_3 + 1$$

$$\boxed{c_3 = -1}$$

$$\text{Power of } L = 0 \Rightarrow a_3 + b_3 - 3c_3 - 1$$

$$\text{power of } T = 0 \Rightarrow -b_3 - 2$$

$$\boxed{b_3 = -2}$$

23)

$$a_3 = -b_3 + 3c_3 - 1$$

$$\begin{aligned} \text{L.H.S.} &= -(-2) + 3(-1) - 1 \\ &= 2 - 3 + 1 \end{aligned}$$

$$\boxed{a_3 = 0}$$

Substituting  $a_3, b_3, c_3$  Value egn ④

$$\bar{\pi}_3 = \left[ l^{a_3} v^{b_3} k^{c_3} \right]$$

$$\bar{\pi}_3 = \left[ l^0 v^{-2} k^1 \right]$$

$$\boxed{\bar{\pi}_3 = \frac{k}{v^2 s}}$$

Sub  $\pi_1, \pi_2$  &  $\bar{\pi}_3$  egn ①

$$f(\pi_1, \pi_2, \bar{\pi}_3) = 0$$

$$f\left(\frac{R}{Jl^2v^2}, \frac{u}{Jlv}, \frac{k}{v^2s}\right) = 0$$

$$\frac{R}{Jl^2v^2} = \left( \frac{u}{Jlv} \cdot \frac{k}{v^2s} \right)$$

$$\boxed{R = Jl^2v^2 \left( \frac{u}{Jlv} \cdot \frac{k}{v^2s} \right)}$$

## \* Dimensionless Numbers & their significance :-

Dimensionless nos are those numbers which are obtained by dividing the inertia force, by Viscous force or gravity force / pressure force or Surface tension force / Elastic force.

As this is a ratio of one force to other force, It will be dimension less number.

These dimensionless numbers are also called Non-dimensional parameters.

- The following are the imp dimensionless no's :-

- 1) Reynolds number ( $Re$ )
- 2) Froude's number ( $Fr$ )
- 3) Euler's number ( $Eu$ )
- 4) Weber's number ( $We$ )
- 5) Mach's number ( $M$ ).

1) Reynolds number ( $Re$ ) :-

It is defined as the ratio of the Inertia force to its Viscous force.

$$\Rightarrow \text{Inertia force} = \text{mass} \times \text{acceleration}$$

$$= \text{Volume} \times \frac{\text{velocity}}{\text{Time}}$$

$$= \frac{dV}{dt} \times \text{velocity} \Rightarrow \rho \times Q \times v \\ = \rho A v \cdot v$$

$$f_i = \rho A v^2$$

$$Re = \frac{f_i}{f_v}, \quad f_v = \text{shear stress} \times \text{Area}$$

$$f_v = \tau A \quad | \quad \tau = \mu \frac{du}{dy}$$

$$f_v = \mu \frac{v}{L} \times A \quad | \quad \frac{du}{dy} = L$$

$$Re = \frac{f_i}{f_v}$$

24)

$$\begin{aligned}
 Re &= \frac{d_i}{\nu} \\
 &= \frac{\rho A V^2}{\mu x v} \text{ area} \\
 Re &= \boxed{\frac{\rho V L}{\mu}} \quad Re = \left( \frac{V L}{\mu / \rho} \right) \quad \left( \frac{\rho}{\mu} = \nu \right) \text{ kinematic viscosity}
 \end{aligned}$$

$$Re = \frac{V L}{\nu}$$

$$\Rightarrow \text{For Pipe flow } Re = \frac{V d}{\nu}$$

For significance:-

This number is taken criterion of dynamic similarity in the flow situation where the viscous force are predominate.

Eg:- Motion of submarine, completely under water.

2) Low velocity motion around automobiles and aeroplanes

3) Incompressible flow pipes are smaller sizes.

2) Froude Number (Fr) :-

It is defined as the square root of the ratio of the inertia force and gravity force.

Mathematically

$$Fr = \sqrt{\frac{F_i}{F_g}}$$

$$F_i = \rho A V^2$$

$$F_g = m \times g$$

$$= V \times A \times g$$

$$= \rho \times L^3 \times A \times g$$

$$F_g = \rho \times A \times L \times g$$

$$F_r = \sqrt{\frac{f A v^2}{f A L g}}$$

$$F_r = \sqrt{\frac{v^2}{L g}}$$

$$\boxed{F_r = \frac{v}{\sqrt{L g}}}$$

Significance :-

Froude number governs the dynamic similarity of the flow situation, where gravitational force is most significant and all other forces are comparatively negligible.

Eg:- Flow over notches & weirs

Flow over a spillway of dam.

Flow through open channels.

3) Euler's number ( $E_u$ ) :-

It is defined as the square root of the ratio of the inertia force to the pressure force.

- Mathematically,  $E_u = \sqrt{\frac{f_i}{F_p}}$

$$\boxed{f_i = f A v^2}$$

$$\boxed{F_p = P \times A}$$

$$E_u = \sqrt{\frac{f_i}{F_p}}$$

$$E_u = \frac{f A v^2}{P \times A}$$

$$E_u = \sqrt{\frac{f v^2}{P}}$$

$$\boxed{E_u = \frac{v}{\sqrt{P/f}}}$$

25)

## Significance

The Euler's number is imp in flow situations in which a pressure gradient exist.

Eg:- Discharge through orifice, and mouth pieces.

- Pressure change due to sudden closed of valves
- Flow through pipes.
- Water hammer created in penstock.

4. Weber number (We) :-

It is defined as the square root of the ratio of inertia force to surface tension force.

$$We = \sqrt{\frac{f_i}{f_s}}$$

$$f_i = \rho A V^2$$

Dimensionless =  $\rho \times L^2$  of unit less unit of

Dimension We =  $\sqrt{\frac{f_i}{f_s}}$  passing through in making

We =  $\sqrt{\frac{\rho A V^2}{\sigma L}}$  dimensionless number called Weber number.

Dimensionless We =  $\sqrt{\frac{\rho \times L \times V^2}{\sigma}}$ . all quantities are converted

$$We = \frac{\rho V}{\sqrt{\frac{\sigma}{\rho L}}}$$

Significance:- This number assumes importance in the following flow situations.

Eg:- 1) Capillary Movement of water in soils,

2) Flow of blood in veins and arteries.

5. Mach's number :-

It is defined as the square root of the

ratio of the inertia force to the elastic force

$$M = \sqrt{\frac{f_i}{f_c}}$$

above to Mach = m

$$f_i = f A v^2$$

$$f_e = k \times A$$

$$M = \sqrt{\frac{f_i}{f_e}} = \sqrt{\frac{f A v^2}{k A}}$$

$$= \sqrt{\frac{f v^2}{k}} = \frac{v}{\sqrt{k}}$$

$$M = \frac{V}{\sqrt{k/g}}$$

Significance :- The mach's number is imp in compressible flow problems at high velocities, such as high velocity of flow in pipes (or) motion of high speed projectiles & missiles.

### \* Similitude :-

To find solution to numerous complicated problems in hydraulic engineering and fluid mechanics model studies are usually conducted. In order that results obtained in the model studies represent the behaviour of prototype, the following three similarities must be ensured between the model and the prototype

1. Geometric similarity

2. Kinematic similarity

3. Dynamic similarity.

1. Geometric similarity :-

For geometric similarity to exist b/w the model and the prototype, the ratios of corresponding length in the model and in the prototype must be same and the included angles b/w two corresponding sides must be the same. Models which are not geometrically similar are known as geometrically distorted model.

Let

$l_m$  = length of model.

$H_m$  = Height of model.

$D_m$  = Diameter of model

$A_m$  = Area of model.

$V_m$  = Volume of model

and  $L_p, B_p, H_p, D_p, A_p$  and  $V_p$  = Corresponding values of the prototype.

Then, for geometric similarity, we must have the relation.

$$\frac{L_m}{L_p} = \frac{B_m}{B_p} = \frac{H_m}{H_p} = \frac{D_m}{D_p} = L_r$$

where  $L_r$  is called the scale ratio or the scale factor

Similarity ratio =  $\frac{A_m}{A_p} = L_r^2$

$$A_r = \text{Area ratio} = \frac{A_m}{A_p} = L_r^2$$

$$V_r = \text{Volume ratio} = \frac{V_m}{V_p} = L_r^3$$

## 2. Kinematic similarity :-

Kinematic similarity is the similarity of motion.

If at the corresponding points in the model and the prototype, the Velocity or acceleration ratios are same and Velocity or acceleration vectors point in the same direction, the two flows are said to be kinematically similar.

Let  $(V_1)_m$  = Velocity of fluid at point 1 in the model.

$(V_2)_m$  = Velocity of fluid at point 2 in the model.

$(a_1)_m$  = Acceleration of fluid at point 1 in the model.

$(a_2)_m$  = Acceleration of fluid at point 2 in the model.

and  $(V_1)_p, (V_2)_p, (a_1)_p, (a_2)_p$  = Corresponding values at the corresponding points of fluid velocity and acceleration in the prototype.

Then for kinematic similarity, we must have.

$$\frac{(V_1)_m}{(V_1)_p} = \frac{(V_2)_m}{(V_2)_p} = V_r \text{ velocity ratio - ①}$$

$$\frac{(a_1)_m}{(a_1)_p} = \frac{(a_2)_m}{(a_2)_p} = a_r \text{ acceleration}$$

→ The directions of the velocities in the model and prototype should be same.

- The geometric similarity is a prerequisite for kinematic similarity.

### 3. Dynamic Similarity:-

Dynamic similarity is the similarity of forces.

The flows in the model and in prototype are dynamically similar if at all the corresponding points, identical types of forces are parallel and bear the same ratio. To dynamic similarity, the force polygons of the two flows can be superimposed by change in force scale.

$(F_i)_m$  = Inertia force at a point

$(F_v)_m$  = Viscous force at the point

$(F_g)_m$  = Gravity force at the point

$(F_i)_p$ ,  $(F_v)_p$ ,  $(F_g)_p$  = Corresponding values of forces at the corresponding point in prototype.

∴ dynamic similarity we have.

$$\frac{(F_i)_m}{(F_i)_p} = \frac{(F_v)_m}{(F_v)_p} = \frac{(F_g)_m}{(F_g)_p} \dots = F_r$$

i.e. the directions of the corresponding forces at the corresponding points in the model and prototype should also be same.

27)

\* Model studies. (or) Model Analysis

This study of model of actual machine or structure is called Model Analysis or Model studies.

Model :-

The model is a small scale replica of the actual structure / machine.

- The actual structure / machine is called prototype.

- In order to know the performance of hydraulic structure & hydraulic machines. Before actually construction / manufacturing them, there are models are made & tested to get the required information.

- The models are not always smaller than the prototype in some cases a model may be even larger / same size as prototype depending upon its need / purposes.

# Types of model.

The hydraulic models are classified as -

1) Undistorted model.

2) Distorted model.

1) Undistorted Model :-

If the scale ratio, for the linear dimensions of the model prototype is same, then the model is said to be "undistorted model".

- Undistorted models are those models which are geometrically similar to their prototype.

- Geometry similarity means have geometric similarity in length, breadth & height & head of water.

## 2) Distorted method :-

A distorted model is said to be a distorted model only when it is not geometrically similar to prototype (or) Models in which the difference scale ratios are used for linear dimensions.

- For eg:- In case of rivers, reservoirs, Harbors etc. two different scale ratios one for horizontal dimensions and for other vertical dimensions are taken.
- Thus the models are rivers, Reservoirs, Harbors will become as a distorted model.

## \* Applications of Dimensional Analysis:-

1) To convert its physical quantity from one system to another.

2) To check the correctness of a physical relation.

3) To obtain relationship among various physical quantities in world.

4) To find dimensions of constant in a physical relation.

## Basics of Turbo machinery (or) Impact of Jet on Vanes.

Analysis & design of turbo machinery is essentially based on the knowledge of forces exerted on (or) by the moving fluids. The forces are caused by a change in the momentum of a jet of fluid.

Impact of jet :- Impact of jet means the force exerted by the jet on the solid body which may be stationary / moving.

Jet :- Jet refers to a stream of fluid emerging from a nozzle.

Vane :-

The vane is a flat / curved plate fixed to the rim of a wheel.

Newton's 2<sup>nd</sup> law :- It's state that the body is rate of change in momentum, is equal to the net force acting on it.

$$f = ma$$

Momentum :-

The quantity of motion of a moving body, measured as a product of its mass & velocity.

$$p = mv$$

Impulse = change in momentum

$$I = F \cdot t$$

Impulse-momentum principle :-

From Newton's 2<sup>nd</sup> law.  $f = ma$  :-

$$f = m \alpha \frac{v_1 - v_2}{t}$$

$$F \cdot t = m v_1 - m v_2$$

$F_{xt} = \text{Initial momentum} - \text{final momentum}$  (38)

$F_{xt} = \text{mass} \times \text{initial velocity} - \text{mass} \times \text{final velocity}$

$f_{xt} = \text{mass} (\text{initial velocity} - \text{final velocity})$ .

Force exerted by the jet on the plate is in the direction of jet.

$$f = ma$$

$$f = m(v_1 - v_2)$$

$$f = \frac{\text{mass}}{t}$$

$$f = \rho \times \text{volume} \cdot \left( \frac{v_1 - v_2}{t} \right).$$

$$f = \rho \times \frac{\text{volume}}{\text{time}} \times (v_1 - v_2)$$

$$f = \rho A (v_1 - v_2).$$

$$F = \rho A v (v_1 - v_2)$$

$$\boxed{F = \rho A v (v_1 - v_2)}$$

→ The forces exerted by the jet on the plate is worked out with the assumptions

1. stationary surface / plate is smooth

2. There is no loss of energy due to impact of jet.

3. Jet will move over the plate after striking with a velocity equal to initial velocity.

4. There is negligible variation in the elevation of the incoming & outgoing jets.

5. There is uniform distribution of velocity throughout.

6. Constant pressure, pressure everywhere is atmospheric.

A) Force exerted by the jet on the necessary plate.

→ Plate is vertical to the jet.

→ Plate is inclined to the jet.

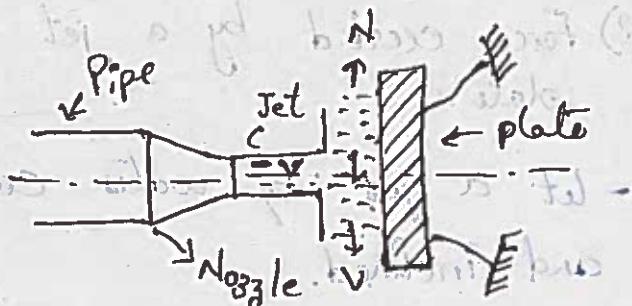
→ Plate is curved.

- 29) B) Force exerted by the jet on the moving plate.  
 → plate is vertical to the jet.  
 → plate is inclined to the jet.  
 → plate is curved.

Force exerted by the jet on a stationary plate :-

- A) i) Force exerted by jet on a vertical plate:

- Consider of jet of water coming out from the nozzle, strikes a flat vertical plate.



Let  $v$  = velocity of the jet.

$d$  = diameter of the jet.

$$a = \text{area of c/s of the jet} = \frac{\pi}{4} d^2$$

- The jet after striking will get deflected to  $90^\circ$  because the plate is at right angles to the jet. Hence the component of the velocity of jet in the direction of jet after striking will be zero.

- The force exerted by the jet on the plate is in the direction of the jet.

$f_x$  = Rate of change in momentum in the direction of force.

$$f_x = \text{mass} \left( \frac{v_1 - v_2}{t} \right)$$

$$f_x = \frac{m v_1 - m v_2}{t}$$

$$f_x = \frac{\text{mass}}{\text{Time}} (\text{initial velocity} - \text{final velocity})$$

$$f_x = P \times \frac{\text{volume}}{\text{Time}} (v_1 - v_2)$$

$$F_x = P A (v_1 - v_2)$$

$$F_x = P A v (v_1 - v_2)$$

$$F_x = P A v (v - 0)$$

$$F_x = P A v^2$$

2) Force exerted by a jet on stationary inclined plate :-

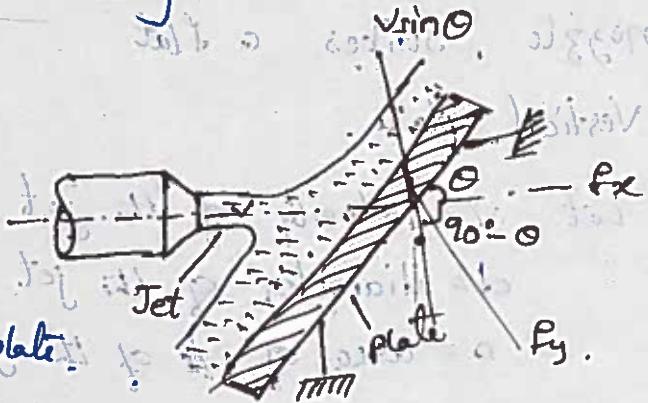
- let a jet of water coming out from the nozzle strikes and inclined.

Let,

$v$  = Velocity of jet in direction of  $x$ .

$\theta$  = angle b/w the jet & plate.

$a$  = Area/s of the jet.



- The mass of water per sec striking the plate =  $f_a v$

- Let us find force exerted by a jet on the plate in the direction normal to the plate.

- Let the force is represented is  $F_n$

$$F_n = \text{mass/sec} (v_1 - v_2)$$

$$= f_a v (v \sin \theta - 0)$$

$$F_n = f_a v^2 \sin \theta$$

- This force can be resolving into two components in direction of the jet and other parallel to flow

$F_x$  Component of  $F_n$  in the direction of flow

Resolving  $F_n$  in two Components.

$$F_x = F_n \cos (90^\circ - \theta)$$

$$F_x = F_n \sin \theta$$

30)

$$F_x = \rho v^2 \sin \theta \times b \sin \theta$$

$$F_x = \rho a v^2 \sin^2 \theta$$

- In  $F_y$  direction

$$F_y = f_n \sin (90^\circ - \theta)$$

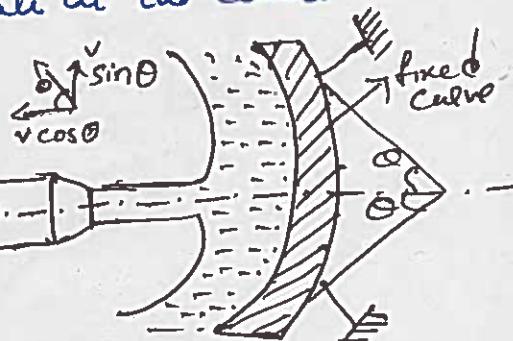
$$= F_n \cos \theta$$

$$F_y = \rho a v^2 \sin \theta \times \cos \theta$$

3. Force exerted by a jet on stationary curved plate :-

1. Jet strikes the curved plate at its centre.

Let a jet of water strikes a fixed curved plate at its centre.



The jet after striking the plate comes out with the same velocity.

- The velocity at outlet of the plate can be resolved into 2 components ; One direction of jet & Other Lateral to the jet

- Component of Velocity in the direction of jet :-

-  $v \cos \theta$  [ - taken as the velocity at outlet is in opposite direction of jet of water coming out from nozzle ] .

- Component of Velocity Lateral to jet

$$F_x = \text{mass/sec} (v_{2x} - v_{1x})$$

$$F_x = \rho a v (v - v \cos \theta)$$

$$\text{Also } v_{2x} = v + v \cos \theta$$

$$F_x = \rho a v^2 (1 + \cos \theta)$$

$$F_y = \text{mass/sec} (v_1 - v_2)$$

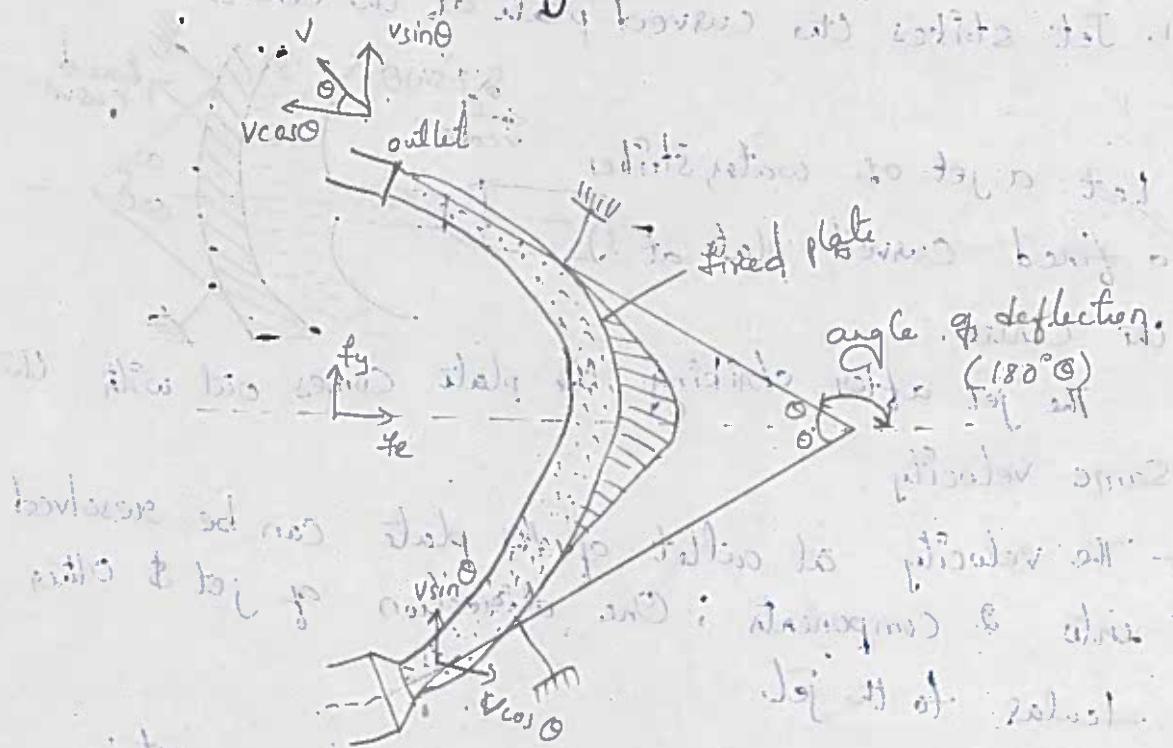
$$= \rho A v (0 - v \sin \theta)$$

$$\boxed{F_y = -\rho A v^2 \sin \theta}$$

Here '-ve' sign means that force is acting in the downward direction.

- In this case the angle of deflection of the jet.

(i) Jet strikes the curved plate at one end tangentially when the plate is symmetrical.



- Let the jet strikes the curved fixed plate at one end tangentially.

- Let the curved plate is symmetrical about x-axis. Then the angle made by the tangents at the two ends of the plate will be same.

Let

$V$  = Velocity of jet of water

$\theta$  = angle made by jet with x-axis, at inlet lip of the curved plate.

The force exerted by the jet of water is in the direction of x & y axis:

$$1) F_x = \text{mass/sec} (v_{1x} - v_{2x})$$

$$= f_{av} (v \cos \theta - (-v \cos \theta))$$

$$= f_{av} (v \cos \theta + v \cos \theta)$$

$$F_x = f_{av} (2v \cos \theta)$$

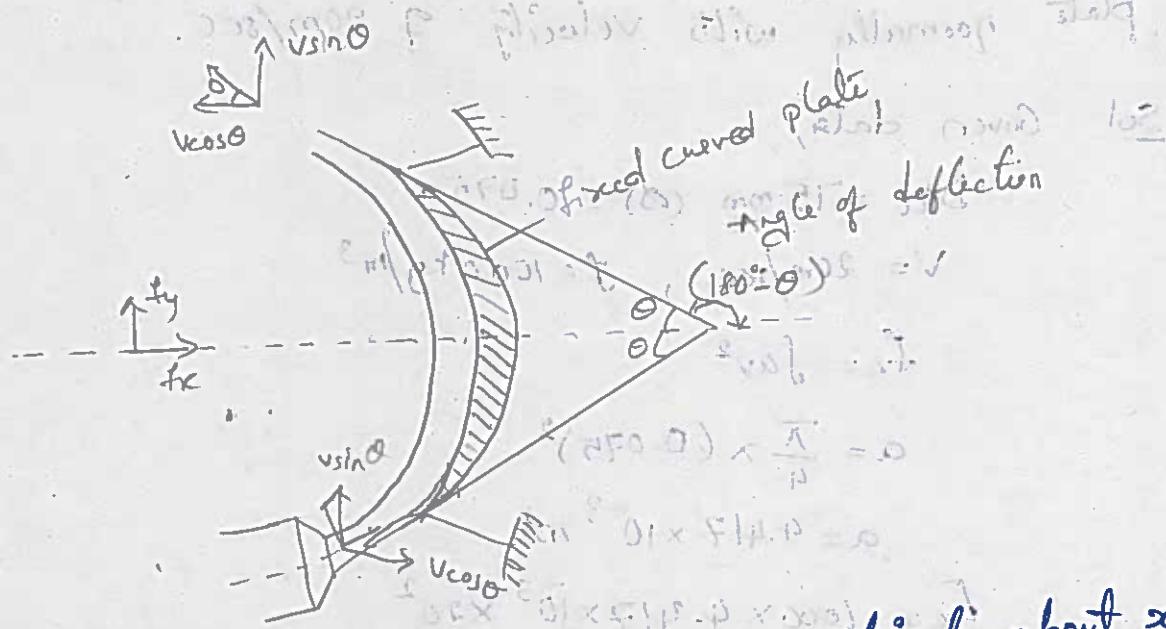
$$\boxed{F_x = 2f_{av}^2 \cos \theta}$$

$$2) F_y = \text{mass/sec} (v_{1y} - v_{2y})$$

$$F_y = f_{av} (v \sin \theta - v \sin \theta)$$

$$\boxed{F_y = 0}$$

- 3) Jet strikes the curved plate at one end tangentially when the plate is unsymmetrical:-



→ When the curved plate is unsymmetrical about x-axis then angle made by its tangents drawn at the inlet and outlet tips of the plate with x-axis will be different.

Let  $\theta$  = Angle made by tangent at inlet tip with x-axis

-  $\phi$  = Angle by the tangent at outlet tip with x-axis

- The force excited by the jet of water is in the direction of  $x$  &  $y$  are

$$F_x = \text{mass/sec} (v_{1x} - v_{2x})$$

$$= f_{av} (v \cos \theta - (-v \cos \theta))$$

$$= f_{av} (v \cos \theta + v \cos \theta)$$

$$\boxed{F_x = f_{av} v^2 (\cos \theta + \cos \theta)}$$

$$F_y = \text{mass/sec} (v_{1y} - v_{2y})$$

$$= f_{av} (v \sin \theta - v \sin \theta)$$

$$\boxed{F_y = f_{av} v^2 (\sin \theta - \sin \theta)}$$

Problem:-

- 1) Find the force exerted by a jet of water dia of 75mm on a stationary flat plate, when the jet strikes the plate normally with velocity of 20m/sec.

Sol Given data,

$$\text{Dia} = 75 \text{ mm } (\varnothing) = 0.075 \text{ m}$$

$$V = 20 \text{ m/sec} , \rho = 1000 \text{ kg/m}^3$$

$$f_x = f_{av} v^2$$

$$a = \frac{\pi}{4} \times (0.075)^2$$

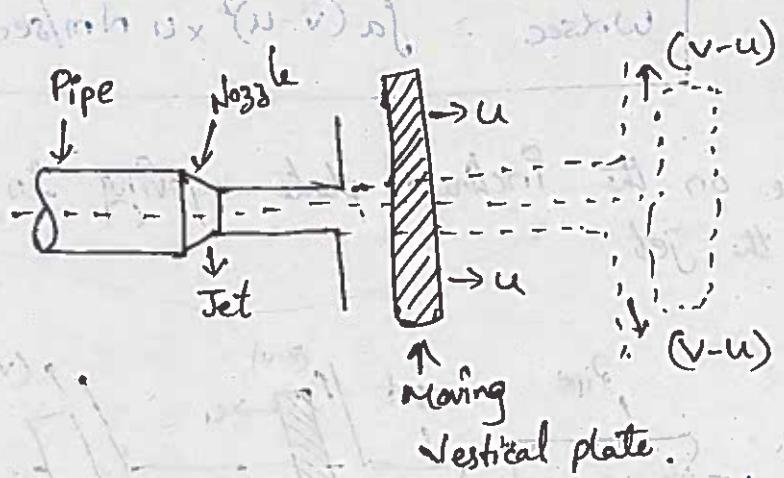
$$a = 4.417 \times 10^{-3} \text{ m}^2$$

$$f_x = 1000 \times 4.417 \times 10^{-3} \times 20^2$$

$$\boxed{f_x = 1766.8 \text{ N}}$$

(2) B) Force exerted by a jet on moving plates:-

- 1) Face on flat vertical plate moving in the direction of jet.



Let a jet of water striking a flat vertical plate moving with a velocity away from the jet.

- let 'v' = velocity of jet.

$a$  = area of c/s of jet.

$u$  = velocity of the flat plate with a

- In this case, the jet doesn't strike the velocity ( $v$ ). but it strikes with a relative velocity - which is equal to the absolute velocity of the jet of water

- The velocity of the plate. Hence, relative velocity of the jet with respect to plate =  $(v-u)$ .

$$F_x = \text{mass/sec} (v - v_e)$$

$$F_x = fa(v-u) [(v-u)-0]$$

$$\boxed{F_x = fa(v-u)^2}$$

- In this case work will be done by the jet on the plate as plate is moving. For the stationary plates the workdone is zero.

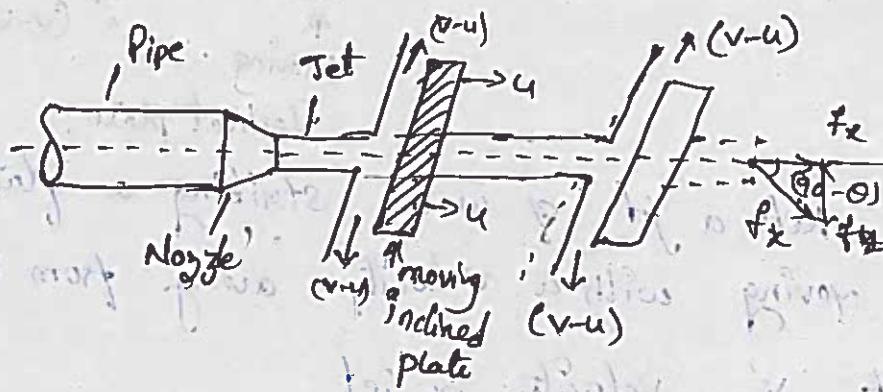
- Workdone per sec by the jet on plate.

$$\text{Work done/sec} = F \times \frac{\text{distance in the direction of force}}{\text{Time}}$$

$$= F_i \times 1 \text{ Nm/sec}$$

$$W_{\text{sec}} = \rho a (v-u)^2 \times u \text{ Nm/sec}$$

2) Force on the inclined plate moving in the direction of the jet.



Let a jet of water strikes an inclined plate which is moving with a uniform velocity in the direction of jet.

Let  $v$  = absolute Velocity of jet of a water.

$u$  = Velocity of the plate.

$\theta$  = Angle b/w jet & plate.

$a$  = c/s area of jet.

- Normal Force ( $F_n$ )

$$F_n = \text{mass/sec} (v-u)$$

$$= \rho a (v-u) [(v-u) \sin \theta - 0]$$

$$F_n = \rho a (v-u)^2 \sin \theta$$

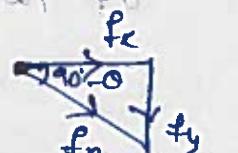
$$F_x = f_n \sin \theta$$

$$= \rho a (v-u)^2 \sin \theta \times \sin \theta$$

$$F_x = \rho a (v-u)^2 \sin^2 \theta$$

$$F_y = f_n \cos \theta$$

$$F_y = \rho a (v-u)^2 \sin \theta \times \cos \theta$$



$$\cos(90-\theta) = \frac{F_x}{F_n}$$

$$F_x = F_n \sin \theta$$

$$\sin(90-\theta) = \frac{F_y}{F_n}$$

$$= f_n \sin(90-\theta)$$

$$F_y = F_n \cos \theta$$

3) Problem

- 1) A jet of water of dia. 10cm strikes a flat plate normally with a velocity of 15m/s. This plate moving with a velocity of 6m/s in the direction of the jet and away from the jet. Find.
- 1) The force exerted by the jet on the plate,
  - 2) Work done by the jet on the plate per sec.

Sol Given data,

$$\text{Jet of water dia } (d) = 10\text{ cm} = 0.1\text{ m}.$$

$$V = 10\text{ m/s}.$$

$$\text{Velocity of moving plate } (u) = 6\text{ m/s}.$$

$$F_x = \rho \alpha (v-u)^2 \\ = 1000 \times \frac{\pi}{4} (0.1)^2 [10^{-6}]^2.$$

$$F_x = 635.35\text{ N}.$$

$$\text{Work done } (W) = F_x \times u$$

$$= 635.35 \times 6$$

$$W = 3812.1 \text{ watts.}$$

$= 0 =$